

## **Elia Fioravanti: Automorphisms of special groups (2)**

Outer automorphism groups can display a wide range of behaviours: from the beautiful structure of arithmetic groups, mapping class groups and  $\text{Out}(F_n)$ , all the way to the near-wildness of automorphisms of certain Baumslag-Solitar groups. To this day, there remain essentially only two general classes of groups whose automorphisms are both well-behaved and well-understood: relatively hyperbolic groups on the one side (based on techniques introduced by Rips and Sela), and virtually polycyclic groups on the other (after Auslander, Baumslag, Baues and Grunewald). Much less is known on automorphisms of non-positively curved groups. The goal of the minicourse will be to motivate why Haglund and Wise's compact special groups should be the "right" place to look for structure. After giving a survey of what is known on their automorphisms, I plan to give a fairly faithful account of the recent result that  $\text{Out}(G)$  is finitely generated for every compact special group  $G$ . I will discuss all the key concepts and techniques that go into the proof: Dehn twists, uniformly WWPD trees, the Bestvina-Paulin construction, the structure of  $R$ -trees arising from automorphisms of special groups, and the "hierarchical" shortening argument.

## **François Dahmani: Conjugacy separability for polynomially growing free-by-cyclic groups**

## **Sasha Bontemps: Subgroup mixing in Baumslag-Solitar groups**

Endowed with the Chabauty topology, the space of subgroups of any infinite countable group  $G$  is a closed subspace of the Cantor set, equipped with an action by homeomorphisms given by the  $G$ -conjugation. We are interested in the dynamics induced by this action on closed  $G$ -invariant subspaces. The largest closed subspace without isolated point is an example of such subspace called the perfect kernel of  $G$ . In an acylindrically hyperbolic context, Hull, Mynasyan and Osin demonstrated strong mixing properties (namely  $\mu$ -mixing for a suitable measure  $\mu$  on  $G$ , a strengthening of high topological transitivity). We uncover a radically different situation in the case of non metabelian Baumslag-Solitar groups. For the decomposition of the perfect kernel introduced by Carderi, Gaboriau, Le Maître and Stalder, who proved high topological transitivity on each piece, we show that the conjugation is even  $\mu$ -mixing in the case of unimodular Baumslag-Solitar groups. On the contrary, when the group is non unimodular, there exists a continuum of measures  $\mu$  for which the action is  $\mu$ -mixing only on a single piece of the partition.

## **Adrien Abgrall: Geometry of outer space for right-angled Artin groups**

Right-angled Artin groups are well-known examples of finitely presented groups acting on CAT(0) cube complexes (median graphs). Some natural choices of actions endow these groups with (pullback) median structures. Outer automorphisms of right-angled Artin groups, and more specifically the ones which coarsely preserve these median structures, have received a lot of interest since the works of Charney-Stambaugh-Vogtmann in 2017. There, the authors construct an Outer space, an explicit classifying space for a subgroup of finite index of this group of automorphisms. I will present generalizations of this result, and rigidity results providing a better geometric understanding of this Outer space.

## Naomi Andrew: Automorphisms behaving badly

Baumslag-Solitar groups are HNN extensions of the infinite cyclic group, whose isomorphism type is controlled by two integers giving the two embeddings. They have provided many counterexamples over the years: for example, they include groups which are not Hopfian and groups which are Hopfian but not residually finite. Later, Collins and Levin showed that there are Baumslag-Solitar groups that do not have finitely generated automorphism group.

Moving this construction to higher rank, one can study "Leary-Minasyan groups": these are HNN extensions of free abelian groups, with both inclusions finite index. They are also sources of counterexamples, such as groups which are CAT(0) but not biautomatic. We study their automorphism groups, and in particular characterise when they are finitely generated; this includes some finitely presented metabelian groups with automorphism groups that are not finitely generated. This is joint work with Sam Hughes and Motiejus Valiunas.